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THE FIBONACCI SEQUENCE: FASCINATING FEATURES AND APPLICATIONS IN SECODARY AND POST SECONDARY SCIENCE AND STEAM EDUCATION

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ABSTRACT:

This paper presents amazing properties of Fibonacci numbers among which is the famous golden ratio. Applications and features of the golden ratio in nature such as Fibonacci spiral in human face, in the ratio of human arm to palm, or within the joints of the palm and other joints within the body. In Arts it emerges in music and in Science and Technology in studying properties of famous quasi-crystal structure and its application in science and engineering; all were been addressed and discussed. Fibonacci chain – the origin of the Fibonacci numbers- was analyzed exploring its application in material science, by studying different properties of quasi-crystals – a newly identified non-periodic crystal material. Its application in photonics and other technological areas were presented and discussed.

KEY WORDS: Fibonacci numbers, Fibonacci chain, Golden ratio, Quasi-crystals, Photonics

1.0 Introduction

Mathematical methods in physics education have become the tools for describing and analyzing physical phenomena (Guida, Lustrac, & Priou, 2003). Take for instance a man swinging his hand while walking; the situation can be studied by observing the ideal situation of a simple pendulum. Amplitude, period, frequency and some other physical factors can be analyzed by means of relevant mathematical tools. In fact, Newtonian mechanics, or to be specific differential equations play crucial role in studying the damping effects of the real situation which leads to

forced vibration and resonance out of which many beautiful technological applications emerges: Just imagine the vibration of pistol and its function in a car engine and other technological devices. At atomic level, researchers have studied the atomic vibration of periodic crystals by means of simple theoretical models. It is known that for periodic models the simplest are linear chain models (Quilichini, 1997).

If $U_n \exp(i\omega t)$ ... ... (1)

Is the displacement of the nth particle in a chain, and the nth and $(n-1)$th particles are connected by a spring with constant α_n, then the equations of motion using Newtonian classical mechanics read

$$M_n\omega^2 u_n = \alpha_n(u_n - u_{n-1}) + \alpha_{n-1}(u_n - u_{n+1}) \ldots (2)$$

Generally speaking, advanced and basic mathematical tools were being in used by theoretical physicists whenever a need for description of a physical phenomenon or a proof of experimental findings.

The application of mathematical series or sequence emerges in science and technology for the past many decades. Most of these series are periodic and deterministic. Examples of these series are, Arithmetic progressions (A.P.), Geometric Progression (G.P.), others are Power series, Fourier series among others. Another class of series is found to be non-periodic or aperiodic but also well deterministic. In the actual sense, this class of sequence has few members compared with the other. Sequences such as Fibonacci sequence, Octonacci sequence, Thue-Morse and Radin Shapiro are the members of this class. This class of aperiodic sequence has not been recognized in theoretical Physics until after the discovery of quasi-crystals in 1984 by Daniel Schetman (Hao, Jie, Ru-Wen, & Wang, 2019). These quasi-crystals are solid materials of long range order but lacks periodicity; a consequence that puzzles the scientific community. The quasi periodic property of this material leads to the display of many abnormal fascinating properties. Researchers found it to be making sense on the course of studying the properties of these quasi-crystals using these aperiodic sequences as theoretical tools (Sudipta, 2017). Aperiodic deterministic sequences have become theoretical Physics tools used to study non periodic crystals (quasi-crystals), for the past three decades. Moreover, their application in material science and technological field is eventually becoming wide and impressive.

Before that in 1984, Scientists believed that solids can only be either crystals or amorphous; on which crystals been characterized by periodic arrangement of points which may be atoms or molecules, long range order and only acceptable 1-fold, 2-folds, 3-folds, 4-folds and 6-folds rotational symmetry. Other symmetries like 5-folds, 8-folds, 10-folds and 12-folds were been termed as forbidden symmetries in solids (Jazbec, 2010)

A one - dimensional theoretical model of this quasi-crystal is the Fibonacci chain, which while it does not feature orientational symmetry illustrates many important properties of quasi-crystals and can be generalized to 2 and 3 Dimensions. Fibonacci chain, which emanated from the study of the dynamics of rabbit populations, consists of a well-defined quasi-periodic sequence of long and short intervals, designated A and C, for Adult and Child in which every year each adult has one child and each child becomes an adult. Starting with a single child "C" and then repeatedly applying the "generation rule" we will have C → A, A →AC. Longer sequences may be generated with the first few sequences as

C→A→AC→ACA→ACAAC →ACAACACA.

The interesting property noted is that each generation is the sum of the two previous generations: ACAAC = ACA ⊕ AC. The frequencies of `A` and `C` in each sequence and the ratio of successive Fibonacci number or number of line segments in each generation is The Golden Mean in the limit of infinite sequence length, τ.

This sequence can also be obtained by successive application of the substitution rule; L → SL and S → L, where we use S and L instead of A and C. The sequence is self- similar, this implies that inflation or deflation of a Fibonacci sequence yields another Fibonacci sequences with different length segment as shown in fig 1.

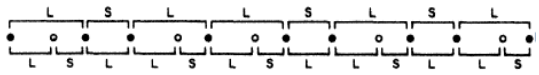

Fig 1: Inflation property of Fibonacci chain

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Furthermore, the diffraction of the Fibonacci sequence gives sharp Bragg peaks with many interesting features. In a one-dimensional Fibonacci quasi-crystal, the longs and shorts could represent the inter-atomic distances; or strength of the bonds between the atoms; or which of two different types of atom is at that position in the chain.

2.0 Amazing Properties of Fibonacci Chain

2.1 Diffraction Patterns

The diffraction pattern of Fibonacci sequence has been well studied using Fourier transform, and found to display sharp Bragg peaks resulting from its long range order; and many other interesting features (Z., Ajay, & Amit, 2013)

2.2 Optical Properties

Experimental measurement of optical transmission of quasi-periodic dielectric layers of SiO_2 and TiO_2 according to Fibonacci Chain has revealed a scaling behavior of the transmission coefficient, which can be considered as experimental evidence of the localization of the light waves. (Hao, Jie, Ru-Wen, & Wang, 2019)

Kohmoto et.al calculated the optical properties using transfer matrix formalism. Results obtained confirmed the self-similarities of the transmission spectra of approximant unit cells of the Fibonacci chain. The results displayed show that the Fibonacci model has high absorbance, high reflectivity and low transmittance coefficient. These observed transmission spectrums were shown to fit experimental results (Guida, Lustrac, & Priou, 2003).

2.3 Electronic properties

A good number of publications reveal tentative results obtained regarding the electronic properties which involve the exponential feature and self-similarities of the Fibonacci chain. The sparseness shown by the electronic density of state provides an insight about the poor conductivity of observed in quasi- crystals (Jazbec, 2010).

2.4 Vibrational and Thermal properties

In a computational study of 1-d Fibonacci sequence; the phonon dispersion curves and the vibrational density of states were computed for the model using An-harmonic lattice dynamics calculations. The phase/sound velocity and the group velocity are found to be $7.197\ \mathrm{ms}^{-1}$and $6.0 \times 10^{22}\mathrm{ms}^{-1}$ respectively, and the dependence of the vibrational properties on thermo-dynamical parameters was investigated. The Debye temperature of the model was found to be 241.44 K, and the lattice heat capacity and conductivity at one-tenth of the Debye temperature are:-$\mathrm{C_v} = 1.33 \times 10^3\,\frac{\mathrm{J}}{\mathrm{mol}} - \mathrm{K};\ \mathrm{and}\mathrm{K_c} = 0.085\ \mathrm{Wb}$. These calculated values established the poor conductivity of quasi-crystals. (Halliru, Godwin, & Olumide, 2019)

3.0 Technological Applications

3.1 Communication Engineering

A new mathematically model of artificial Fibonacci quasi-crystal of transmission-line resonators was constructed. The resonator is coupled by capacitors with capacitances $\mathrm{C_L}$ and $\mathrm{C_R}$ on the left and right to the circuits L and R, respectively, which elements are of circuit Quantum Electrodynamics lattices. The variation of wave speeds at impedances results in the gapless frequency spectrum. The Eigen-modes have constant amplitude but have wavelengths that vary according to the Fibonacci quasi-crystal sequence (Moy, 2018).

3.2 Fiber optics

A study of resonant transmission of light using transfer matrix and the analysis of the resonant wavelength and the quasi-periodic structure reveal closed analytical expressions for the transmission coefficient and its application in optical micro-cavities (Macia, 1998). Several device designs have already been demonstrated and include micro-optical circuits, lasers, light emitting diodes, switches, and wavelength division multiplexers (Hao, Jie, Ru-Wen, & Wang, 2019).

4.0 Fibonacci and Science Education

4.1 Introduction

Fibonacci numbers; named after Leonardo Pisano nicknamed as Fibonacci; meaning son of Bonacci (Quilichini, 1997), are sequence originated by studying the generation of rabbits.

The sequence reads: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 . . . . . F_n

The terms in this sequence are obtained by simply adding the preceding two, starting with $F_1 = F_2 = 1$.

Thus in general

$$F_n = F_{n-1} + F_{n-2}$$

Or

$$F_{n+1} = F_n + F_{n-1}$$

with $F_0 = F_1 = 1$ 3.21

For the infinite Fibonacci chain, the ratio between the frequencies will therefore be

$$\lim_{n\to\infty} \frac{F_{n-1}}{F_{n-2}} = \tau = \frac{1+\sqrt{5}}{2} = 1.618$$

Hence, the sequence is perfectly ordered, but its generating rule has nothing to do with periodicity.

The symbolical analog of the Fibonacci sequence, constructed by using two types of building blocks say, A and B can be obtained from the substitution rule B→AB and A→B, whose successive application generates the sequence of letters A, AB, ABA, ABAAB, ABAABABA, [i.e. 1,2,3,5,8,...] and so on (Enrique, 2006).

This sequence has many interesting and miraculous properties that can be seen in our everyday life.

4.1 Basic Computations

Many basic computations using Fibonacci sequence were been explored by (Sudipta, 2017) among which are: -

i. Consider Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 . . . . . .

Take 2, 3 and 5 for example; 2 is a multiple of the next third number (8) 3 is a multiple of the next fourth number (21), 5 is a multiple of the next fifth number (55)……

ii. By taking any two consecutive numbers from the series say 13 and 21, the numbers give the approximate conversions from miles to kilometers and the converse;

13 km = 21 miles; 21km = 35 miles, e.t.c. (Sudipta, 2017)

Noting that the product $F_1 \times F_2 = 1 \times 2 = 2$ can be represented as $a_1F_1^2 + a_2F_2^2$ where $a_1 = a_2 = 1$. This idea motivated (Narayanan, Ramanan, Srikanth, & Likitha, 2019) explore their expertise in group theory, where they generalize it for the product of k- consecutive Fibonacci numbers. Their computational work reveals that: . The product of K consecutive Fibonacci numbers can be expressed as the sum of K^{th} power of Fibonacci numbers.

i.e. for the products

$$F_n \times F_{n-1} \times F_{n-2} \dots \dots \dots F_{n-k+1} = u_1F_1^k + \cdots \dots + u_{n-k}F_{n-k}^k$$

Apart from the extension of this result, its proof and that of other related formulae were been developed and discussed. (Narayanan, Ramanan, Srikanth, & Likitha, 2019)

According to (Gosai, 2019) Binet in 1943 has been able to derive the formula for calculating the nth Fibonacci number. The formular reads

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

4.2 The golden ratio

Fibonacci numbers to converge to a number 1.618 called golden ratio φ. This number is obtained by taking the ratio: $\lim_{n\to\infty} \frac{F_{n+1}}{F_n}$.

The figure 4.2.1 depicts the convergence obtained by plotting the ratio against the Fibonacci numbers. (Ibeh, Halliru, & Ige, 2020)

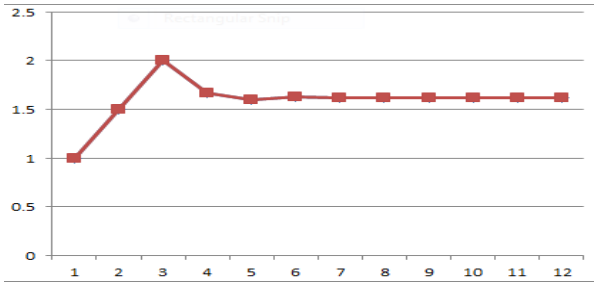
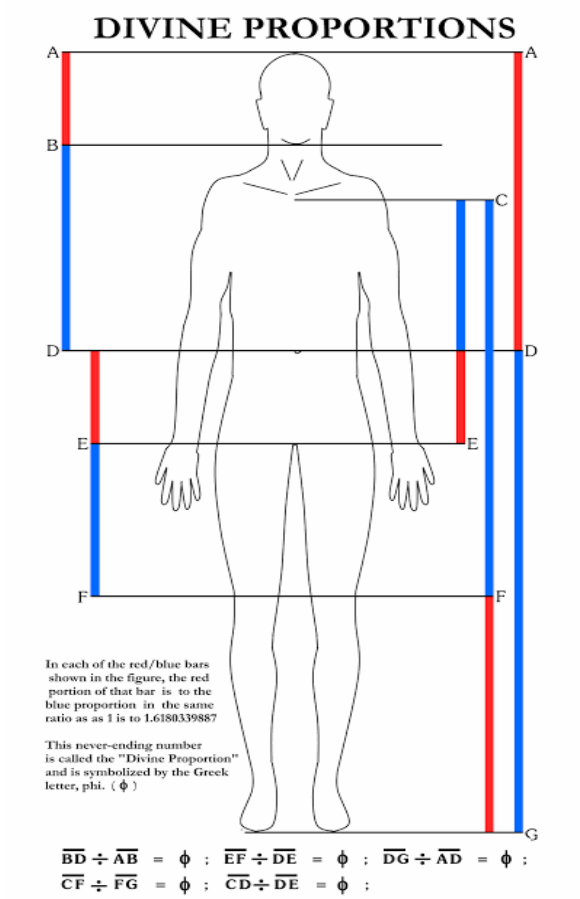


Fig 4.2.1 Convergence of Fibonacci numbers

4.2.1 Golden ratio in nature

The emergence of this golden ratio in nature can be seen everywhere. The following capture clarifies a lot of the amazing properties and applications; in the height of a human being, his hand, his fingers, and his face among others.



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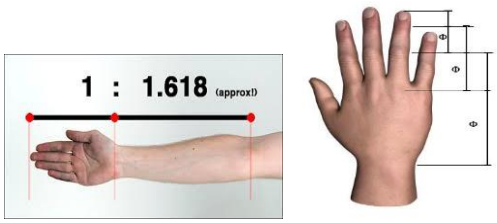
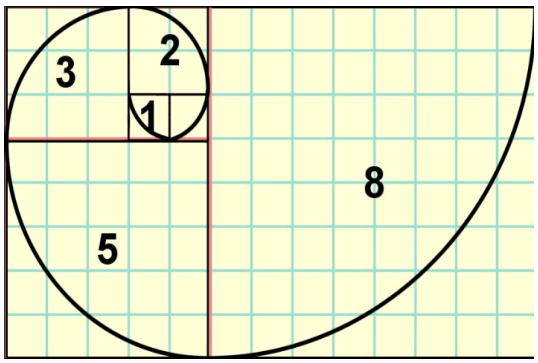


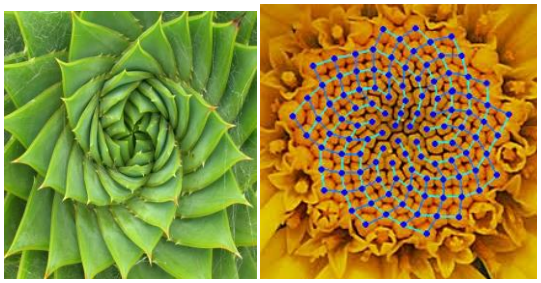
4.2.2 Golden ratio and the Fibonacci spiral

Fibonacci spiral is obtained by drawing squares of the Fibonacci numbers, starting with 1,1, then 2 at the top, 3 by the right, then 5 at the bottom, 8 to the left, 13 at the top, 21 by the right, like that.



Fibonacci Spiral | Animated video YouTube" https://m.youtube.com/watch?v=KKM0dzgJgf8#

This spiral can be seen in many uncountable species in life, in ancient and present art works, in geometry and architecture.



Fibonacci spiral are also found in various fields associated in nature. It is seen in snail, sea shells, waves, combination of colors; roses etc in so many things created in nature. But very few of us have time to study this phenomenon

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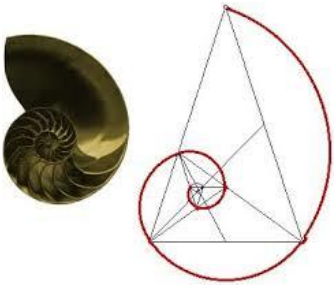

Recent development reveals the application of Fibonacci numbers in science and engineering. In communication, it is found to play a vital role in security coding and in nano technology such as photonic band gaps in opto-electronic devices (Sudipta, 2017)

5.0 Fibonacci sequence in Arts and Trade

As presented in this paper, application of Fibonacci sequence can be found everywhere in nature; in geometry, in art and architecture.

In Music, it is found that in a piano, the intervals between keys are Fibonacci numbers (Sudipta, 2017).

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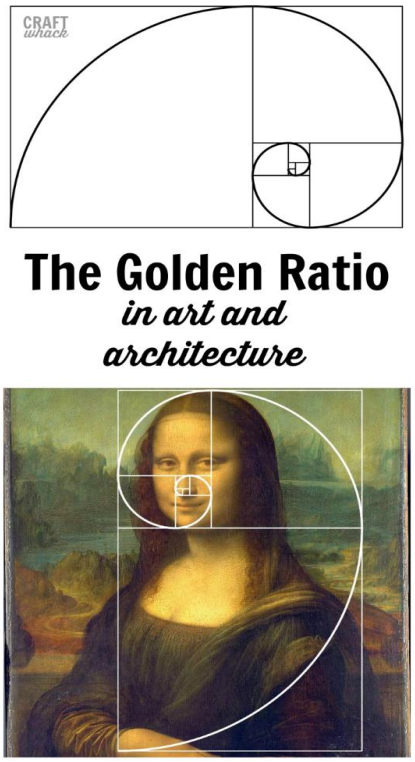

5.1 Fibonacci Retracement and Extension Levels

Retracement levels: This theory monitors the circulatory and trace-back movement of price. So, many traders use retracement levels as potential support and resistance areas. The retracement levels are: - 0.236, 0.382, 0. 618, 0.764. On the other hand the extension levels are: - 0, 0.382, 0. 618, 1.000, 1.382, 1.618. Traders use the Fibonacci extension levels as profit-taking levels. Most of the trade charting software includes both the retracement and extension level tools.

6.0 Python codes for Fibonacci sequence

CODE 1: FIBONACCI NUMBERS

```
defFibonacciStep(n):
# n can only takes a value of Fibonacci number
step = 1
prev = 1
curn = 1
for i in range(0,n):
temp = curn
curn = prev + curn
prev = temp
if(curn == n ):
```

```
return step
step += 1
steps = FibonaciStep(n)
CODE 2 FIBONACCI CHAIN
defgenerateLS(c,t):
    count = c-1
    if(t=="num"):
        L = "1"
    else:
        L = "L"
    if(t=="num"):
        S = "0"
    else:
        S = "S"
    bucket = L+S
    if(c == 0):
        return bucket
    result = ""
    for j in range(0,count):
        result = ""
        for i in range(0,len(bucket)):
            if(bucket[i]==L):
                result += L+S
            else:
                result += L
        bucket = result
    return bucket
```

Conclusion

Looking at the origin of Fibonacci numbers, which is an initiative attempt to solve a hypothetical growth of rabbit based on idealized assumptions, fosters what are exactly needed; identification of real life problems and coming up with the solutions. This inquiry pedagogy adopted by Fibonacci has stirred the motive of Europe scientific community that leads to conducting research for some years. two papers were published; Science education now: a renewed pedagogy for the future of Europe published 2007, the other one reads Critical reflection published in 2008. Finally, the research leads to establishment of enquiry based education projects called Fibonacci projects in the 2009.The goal is to develop technological skills and attitudes for future industrial society (Wyune & pierre, 2019).

In a recent development, students from the polytechnic Institute of New York University constructed a robot that moves based on the Fibonacci sequence (Narayanan, Ramanan, Srikanth, & Likitha, 2019). This advancement is in line with the objectives behind the Science Technology Engineering Art and Mathematics (STEAM) Education.

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